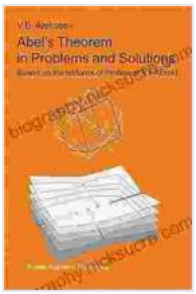
Abel's Theorem: A Comprehensive Guide to Understanding and Applying Its Concepts in Mathematics

Abel's Theorem, named after the renowned Norwegian mathematician Niels Henrik Abel, is a cornerstone of mathematical analysis. It provides a powerful tool for determining the convergence or divergence of series and plays a significant role in various branches of mathematics, including Calculus, Number Theory, and Real Analysis.



Abel’s Theorem in Problems and Solutions: Based on the lectures of Professor V.I. Arnold (Kluwer International Series in Engineering & Computer Scienc)

by V.B. Alekseev

★★★★★ 5 out of 5

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History

Abel first formulated his theorem in 1826 while studying the convergence of power series. He published his findings in a paper titled "Recherches sur les séries infinies," which had a profound impact on the field of mathematics.

Statement of Abel's Theorem

Abel's Theorem states that if:

1. a_n

   is a sequence of real numbers

2. b_n

   is a monotonic sequence of real numbers (either increasing or decreasing)

3. a_n

   is bounded

Then the series

$$\sum a_n(b_n - b_{n+1})$$

converges.

Applications

Abel's Theorem finds numerous applications in mathematics, such as:

- Determining the convergence or divergence of series
- Evaluating infinite series
- Solving differential equations
- Understanding the behavior of functions

- Approximation theory

Problem-Solving Techniques

To apply Abel's Theorem in problem-solving, follow these steps:

1. **Identify the sequences**

$$a_n$$

and

$$b_n$$

in the given series. 2. **Check if the sequence**

$$b_n$$

is monotonic and the sequence

$$a_n$$

is bounded. 3. **If the conditions of Abel's Theorem are met**, conclude that the series converges. Otherwise, it may diverge.

Proof of Abel's Theorem

The proof of Abel's Theorem involves several steps:

1. **Represent the series using partial sums**:

$$S_n = a_1(b_1 - b_2) + a_2(b_2 - b_3) + ... + a_n(b_n - b_{n+1})$$

2. **Telescoping sum**: The terms

$$b_i$$

cancel each other out, leaving only the first and last terms:

$$S_n = a_1b_1 - a_{n+1}b_{n+1}$$

3. **Apply the limit test**: Since

a_n

is bounded and

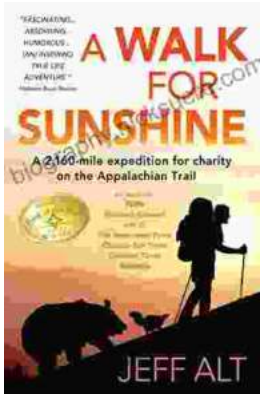
b_n

is monotonic, the limit of

$a_n b_n$

exists. Therefore, the series converges by the limit test.

Abel's Theorem is a powerful and versatile tool in mathematical analysis. Its applications extend to various areas of mathematics, including Calculus, Number Theory, and Real Analysis. By understanding the statement, applications, and problem-solving techniques associated with Abel's Theorem, mathematicians can leverage it effectively to solve complex problems and advance their mathematical knowledge.



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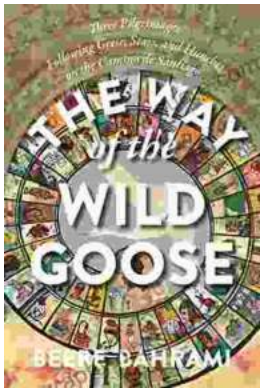

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