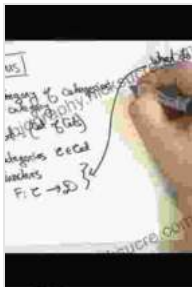# Elementary Overview of Mathematical Structures

Mathematical structures are abstract objects that are used to model and study real-world phenomena. They provide a way to organize and understand the underlying patterns and relationships within a system. In this article, we will provide an elementary overview of some of the most common mathematical structures, including sets, relations, functions, groups, rings, and fields.

Elementary Overview Of Mathematical Structures, An: Algebra, Topology And Categories by Marco Grandis

★★★★★ 5 out of 5

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Enhanced typesetting	: Enabled
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Sets

A set is a collection of distinct objects. The objects in a set are called *elements*. Sets are represented using curly braces, and the elements of a set are separated by commas. For example, the set of natural numbers less than 5 is written as follows:

$$S = \{ 1, 2, 3, 4 \}$$

Sets can be used to represent a wide variety of real-world phenomena, such as the set of students in a class, the set of books in a library, or the set of employees in a company.

Relations

A relation is a set of ordered pairs. The elements of a relation are called *tuples*. Relations are represented using angle brackets, and the tuples in a relation are separated by commas. For example, the relation "is greater than" on the set of natural numbers is written as follows:

$$R = \{ (1, 2),(1, 3),(1, 4),(2, 3),(2, 4),(3, 4) \}$$

Relations can be used to represent a wide variety of real-world phenomena, such as the relation "is a friend of" on the set of people, or the relation "is a subset of" on the set of sets.

Functions

A function is a relation that assigns each element of a set to a unique element of another set. The set of elements that the function assigns to is called the *range* of the function. The set of elements that the function takes as input is called the *domain* of the function. Functions are represented using arrows, and the domain and range of a function are written next to the arrow. For example, the function that assigns each natural number to its square is written as follows:

$$f: \mathbb{N}\rightarrow \mathbb{N}$$

$$f(n) = n^2$$

Functions can be used to represent a wide variety of real-world phenomena, such as the function that assigns each day of the week to the number of hours of sunlight that day, or the function that assigns each student in a class to their grade.

Groups

A group is a set together with an operation that combines any two elements of the set to produce a third element of the set. The operation is often called the *group operation*. Groups are represented using the following notation:

$$(G, \cdot)$$

where G is the set and $\cdot$ is the group operation.

Groups can be used to represent a wide variety of real-world phenomena, such as the group of integers under addition, the group of real numbers under multiplication, or the group of symmetries of a square.

Rings

A ring is a group together with a second operation that distributes over the group operation. The second operation is often called the *ring operation*. Rings are represented using the following notation:

$$(R, +, \cdot)$$

where R is the set, $+$ is the group operation, and $\cdot$ is the ring operation.

Rings can be used to represent a wide variety of real-world phenomena, such as the ring of integers, the ring of polynomials, or the ring of matrices.

Fields

A field is a ring in which every non-zero element has a multiplicative inverse. Fields are represented using the following notation:

$$(F, +, \cdot)$$

where F is the set, $+$ is the group operation, and $\cdot$ is the ring operation.

Fields can be used to represent a wide variety of real-world phenomena, such as the field of rational numbers, the field of real numbers, or the field of complex numbers.

Mathematical structures are a powerful tool for modeling and studying real-world phenomena. They provide a way to organize and understand the underlying patterns and relationships within a system. In this article, we have provided an elementary overview of some of the most common mathematical structures, including sets, relations, functions, groups, rings, and fields. These structures can be used to represent a wide variety of real-world phenomena, and they provide a basis for the study of many areas of mathematics, including algebra, analysis, and geometry.
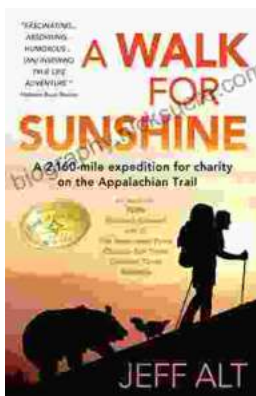
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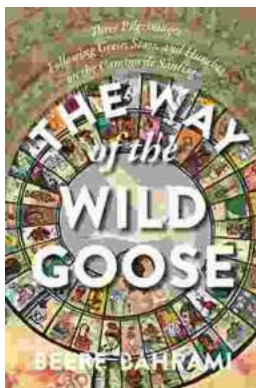Print length : 528 pages

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